

DETERMINING THE CONVECTIVE HEAT TRANSFER  
COEFFICIENT BY THE "HALF-SPACE PERIOD" METHOD

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The method of determining the convective heat transfer coefficient by the "half-space period", based on the laws of heat propagation through a semiinfinite body, is described and checked out experimentally.

The best known and most widely used today methods of measuring the convective heat transfer coefficient  $\alpha$  are:

1. Measurement of the temperature profile at each cross section of the stream and its subsequent differentiation at the channel wall, in accordance with the Nusselt procedure for heat transfer in fluid boundary layers:

$$\alpha(\bar{T}_f - T_w) = -\lambda_f \left. \frac{\partial T_f}{\partial n} \right|_w \quad (1)$$

2. Measurement of the temperature profile across the channel wall and its subsequent differentiation at the channel surface, with  $\bar{T}_f$  calculated from the heat balance and with  $T_w$  measured:

$$-\lambda \left. \frac{\partial T}{\partial n} \right|_w = \alpha(\bar{T}_f - T_w) \quad (2)$$

3. Measurement of the thermal flux density  $q$  at the wall and of the corresponding temperature difference  $\bar{T}_f - T_w$  between fluid and wall:

$$q = \alpha(\bar{T}_f - T_w) \quad (3)$$

4. Application of the laws of the regular heating mode.

Using the first method is always difficult, because of the need for an accurate temperature determination in the boundary layer. The second method, which is known in the technical literature as the gradient method [2], requires not only an accurate determination of the temperature gradient across the wall near the heat transfer surface, but also involves a large amount of computations. The other two methods are often not very feasible, especially in the case of high-temperature gases flowing at supersonic velocities through section-shaped channels.

Here will be described a method of determining the local heat transfer coefficient on the basis of the laws of heat propagation through a half-space.

It is well known [2] that the temperature field in a semiinfinite body under boundary conditions of the third kind is described by the relation

$$\Theta = \operatorname{erfc} \frac{1}{2\sqrt{\operatorname{Fo}_x}} - \exp(\operatorname{Bi}_x + \operatorname{Bi}_x^2 \operatorname{Fo}_x) \operatorname{erfc} \left( \frac{1}{2\sqrt{\operatorname{Fo}_x}} + \operatorname{Bi}_x \sqrt{\operatorname{Fo}_x} \right) \quad (4)$$

As a model of a semiinfinite body can serve a semiinfinite rod with a perfectly insulated lateral surface and facing the stream of fluid with its end. As a model can also serve, however, a rod of finite length with a perfectly insulated lateral surface and its one end shielded from the stream, when the laws

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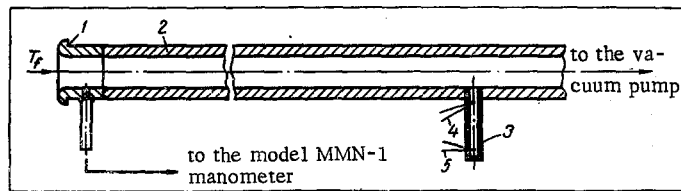


Fig. 1. Basic diagram of the test apparatus: 1) inlet collector, 2) pipe, 3) thermally insulated rod, 4) operating thermocouple, 5) control thermocouple.

of heat propagation through a half-space will evidently remain valid as long as the temperature perturbations have not reached the thermally shielded end. If the unshielded end of such a rod is converted into a channel for the test stream of liquid or gas, then the Biot number  $Bi_x$  or the coefficient  $\alpha$  can easily be determined according to formula (4) on the basis of known properties  $a_1$ ,  $\lambda_1$  of the rod at the test temperature  $T_f$  and at the initial rod temperature  $T_0$  and on the basis of the time  $\tau$  within which a point at a distance  $x$  from the rod end in the stream reaches any temperature  $T(x, \tau)$ .

In order to perfect this method, the authors measured the coefficient of convective heat transfer under classical conditions of fluid flow through a circular pipe after hydrodynamic and thermal stabilization.

The test apparatus (Fig. 1) consisted of a pipe, inside diameter  $d = 0.02$  m and length  $L = 3.0$  m, through which air was passed by means of a vacuum pump. The air flow rate was measured with a model MMN-1 micromanometer at the inlet collector cut along circular arcs according to TsAGI specifications. Through the pipe wall at a distance  $l/d > 50$  from the entrance section was inserted a rod of grade 1Kh18N9T steel ( $\lambda_1 = 15.08$  W/m $\cdot$ °C,  $a_1 = 0.385 \cdot 10^{-5}$  m<sup>2</sup>/sec), diameter  $d_1 = 0.005$  m and length  $l_1 = 0.085$  m, with its uninsulated end flush against the inside pipe surface and exposed to the air stream. On this rod, at a distance  $x = 0.021$  m from that uninsulated end, was stuck a copper-constantan thermocouple whose readings were recorded by a model ÉPP-09 potentiometer. The other end of this rod was thermally insulated and another copper-constantan thermocouple installed here for control, to establish when the conditions of heat propagation would cease to be those in a semiinfinite body.

The measurements were made in a subsonic turbulent air stream with the Reynolds number  $Re = 1.79 \cdot 10^5$  ( $w = 121$  m/sec) and temperature  $\bar{T}_f = -2^\circ\text{C}$ ; the initial rod temperature was  $T_0 = +15.6^\circ\text{C}$ . The temperature trend with time, as recorded by the operating thermocouple, is shown in Fig. 2.

On the basis of the test results, we calculated the convective heat transfer coefficient according to formula (4) at various instants of time  $\tau$  and found that, as long as the laws of heat propagation through a semiinfinite body prevailed ( $\tau_2 \approx 270$  sec), the value of  $\alpha$  remained steady and equal to 326 W/m<sup>2</sup>·°C (Fig. 3). The test results could be very well repeated.

After temperature perturbations had reached the insulated end of the rod ( $\tau \gg \tau_2$ ), the conditions of heat propagation ceased to be those in a semiinfinite body and formula (4) yielded too high values for  $\alpha$ .

The test values of the convective heat transfer coefficient were compared with calculations for a stable flow through a pipe according to the well known formula

$$Nu_f = 0.023 Re_f^{0.8} Pr_f^{0.43} C_t, \quad (5)$$

with the temperature factor assumed  $C_t = 1$  because of the small temperature difference between channel wall and stream. For our test conditions the measured values of  $\alpha$  differed from the calculated values  $\alpha_{calc} = 334$  W/m<sup>2</sup>·°C by not more than 5%. We note, however, that our calculation of the convective heat transfer coefficient was based on the difference between the mean-integral (over a section) stream temperature  $\bar{T}_f$  and the wall temperature  $T_w$ , while the test values of  $\alpha$  were based on the difference between temperatures  $\bar{T}_f$  and  $T_w$ .

The test apparatus was designed so as to eliminate two obvious factors which would invalidate the fundamental relation (4):

- 1) the heat loss through the insulation of the heat absorbing rod;
- 2) deviations from conditions of heat propagation through a semiinfinite body on account of the finite rod length.

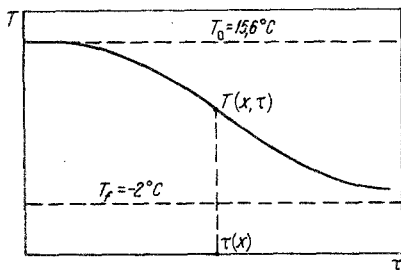


Fig. 2. Temperature trend with time, at any point along the rod.

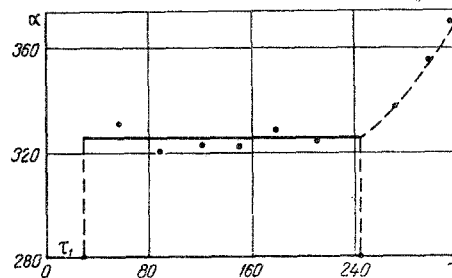


Fig. 3. Time distribution of test values for  $\alpha$  ( $\text{W}/\text{m}^2 \cdot ^\circ\text{C}$ ). Time  $\tau$  (sec).

As is well known, an installation of a thermocouple on a heat absorbing rod surface directly is cumbersome in test practice and does not eliminate the effect of heat leakage through the insulation on the results of measurements. Our analysis of the transient temperature field in a composite cylinder with heat entering at the end face has shown that the heat loss is a function of the following complex groups:

$$Fo_x, Bi_x, \frac{x}{d_1}, \frac{\alpha\tau}{d_1^2}, \frac{\lambda_1}{\lambda_2} \cdot \frac{(d_2 - d_1)}{d_2 \ln \frac{d_2}{d_1}}.$$

The feasibility of evaluating the extent to which the finite length of a test distorts the measurements has been established by testing rods of various lengths. In addition to the rod described earlier, we also tested rods  $d_1 = 0.004$  m in diameter and  $l_1 = 0.05, 0.10, 0.15,$  and  $0.20$  m long made of grade 08 steel with ceramic coating as thermal insulation. Calculations have shown in each case that the effect of heat leakage and of finite rod length can be eliminated as follows:

a) For a determination of  $\alpha$  according to (4), one must use several (5-6) measured values of  $T(x, \tau)$  and  $\tau_1$ , constant values of  $\alpha$  indicating adequately enough that the conditions of heat propagation through a semiinfinite rod have been rather well maintained (Fig. 3).

b) Noticeable deviations of calculated  $\alpha$  values from those constant measured values indicate that the laws of heat propagation through a half-space have ceased to apply.

c) When the control thermocouple begins to read, with a test rod of any length and with any form of thermal insulation, it indicates that the experiment must be terminated (this is the basic function of that thermocouple).

We note that a test rod is made of a material whose thermophysical properties are almost independent of the temperature.

The described method of determining the heat transfer coefficient is based on the assumption, as appears from the preceding discussion, that the temperature at the stream axis  $T_f$  is known. In some cases, however, it is difficult to measure  $T_f$ . It then becomes worthwhile to modify the experiment somewhat. In the heat absorbing rod are installed not one but, for example, two operating thermocouples at respective distances  $x_1$  and  $x_2$  from the rod end which is exposed to the stream. Having measured the time  $\tau_1$  and  $\tau_2$  in which each of these two points has reached some arbitrary temperature  $T(x_1, T_1) = T(x_2, T_2)$ , one can determine both the true value of  $\alpha$  and the parameter  $\theta = (T(x, \tau) - T_0) / (T_f - T_0)$ , i. e., the stream temperature  $T_f$  when the initial rod temperature  $T_0$  is known.

#### NOTATION

- $\alpha$  is the convective heat transfer coefficient;  
 $a_1, \lambda_1$  are the thermal diffusivity and thermal conductivity of the rod material;  
 $\lambda_f$  is the thermal conductivity of the fluid;  
 $\lambda_d$  is the thermal conductivity of the rod insulation;  
 $\lambda_2$  is the outside diameter of the rod insulation;  
 $q$  is the thermal flux density;  
 $d, L$  are the inside diameter and length of the pipe;

$d_1, l_1$	are the diameter and length of the rod;
$x$	is the coordinate measured from the boundary surface of the half-space;
$\tau$	is the time;
$T(x, \tau)$	is the instantaneous temperature;
$\theta$	is the dimensionless instantaneous temperature;
$T_0$	is the initial rod temperature;
$T_f$	is the stream temperature at the channel axis;
$\overline{T}_f$	is the mean-integral (over a cross section) stream temperature;
$T_w$	is the temperature of the channel wall;
$w$	is the mean (with respect to flow rate) stream velocity;
$Re_f$	is the Reynolds number;
$Pr_f$	is the Prandtl number;
$Nu_f$	is the Nusselt number;
$Bi_x = \alpha x / \lambda$	is the local Biot number;
$ Fo_x = a \tau / x^2$	is the local Fourier number.

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